

# Co-viability for the management of fisheries

Main ideas

...

... a little bit of  
mathematics ...

...

...

and some examples

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# Viability

Viability ??



# What about fisheries ?

## Management renewable resources

- **Interdisciplinary:**  
Economy-Ecology-Social
- **Complexity, Nonlinearity**

## New concepts

- **Sustainability**
  - **Reconciliation**  
Economy-Ecology



- **Intergenerational equity**
- **Resilience**
- **Precaution**

# Co-viability for a stylized bio-economic model

- An exploited population dynamics

$$x(t+1) = f\left(x(t) - c(t)\right), \quad t = 0, 1, \dots, +\infty$$

with catches

$$0 \leq c(t) \leq x(t)$$

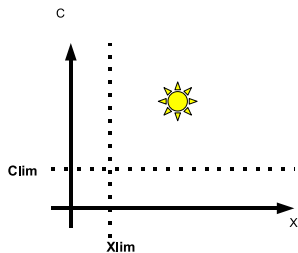
## The co-viability constraints

- A conservation requirement:

$$x(t) \geq x_{\text{lim}}, \quad t = 0, 1, \dots, +\infty$$

- A direct use value:

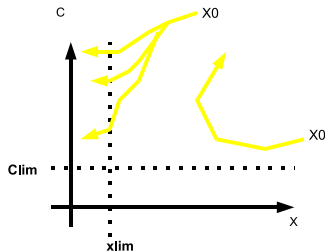
$$c(t) \geq c_{\text{lim}}, \quad t = 0, 1, \dots, +\infty$$



# The viability kernel

## A feasibility set

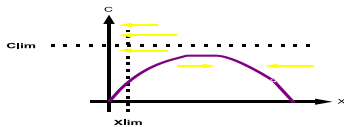
$$\text{Viab} = \left\{ x_0 \mid \begin{array}{l} \exists c(t) \text{ and } x(t) \\ \text{starting from } x_0 \\ \text{satisfying} \\ \text{dyna. + constraints} \\ \text{for any time } t \in \mathbf{R}^+ \end{array} \right\}$$



# A ceiling guaranteed catch & a floor stock

Assume  $f$  increasing;  $f(0) = 0$

No viability



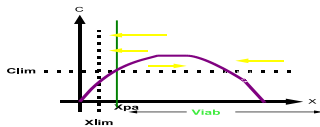
$$Viab = \emptyset$$

- If  $c_{lim} > c_{MSY}$   
where Maximum Sustainable Yield

$$c_{MSY} = \max_{x \geq 0, f(x-c)=x} c$$

- A ceiling guaranteed catch

Partial viability



$$Viab = [x_{pa}, +\infty[$$

- if  $0 \leq c_{lim} \leq c_{MSY}$
- with the precautionary threshold

$$x_{pa} = \min \left( x, x \geq c_{lim}, x \geq x_{lim} f(x-c_{lim}) \geq x \right)$$

- Viability margin:  $x_{pa} > c_{lim}$

## The viable quotas corridor

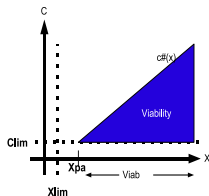
- Method: Maintaining  $x$  in  $Viab$
- Assume  $0 \leq c_{lim} \leq c_{MSY}$ .  
Then Viable Quotas are

$$C_{Viab}(x) = [c_{lim}, c_{\#}(x)]$$

where  $c_{\#}(x) = x - x_{pa} + c_{lim}$

- Flexibility:

- Conservative  $c_{lim}(x)$
- Greedy  $c_{\#}(x)$
- Trade-off:  $\alpha c_{lim}(x) + (1 - \alpha)c_{\#}(x)$
- More efficient strategy: NPV, maximin, Chichilnisky, ...



# An illustration from nephrops in the Bay of Biscay

Martinet-Thébaud-Doyen, 2007

- Dynamics: Beverton-Holt:

$$f(x) = \frac{Rx}{1 + Sx}, \quad R = 1.78, \quad K = 30800$$

- Rent  $\Pi(x, h) = ph - c \frac{h}{qx}$  with

$$p = 8500 \text{ €} \cdot \text{ton}^{-1}, \quad c = 377 \text{ €} \cdot \text{day}^{-1}, \quad q = 72 \cdot 10^{-7} \text{ day}^{-1}$$

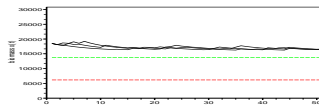
- Co-viability constraints:

$$x_{lim} = 6160 \text{ tons}$$

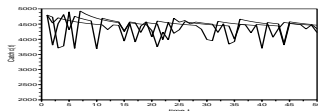
$$\Pi_{lim} = 19500 \text{ Keuros}$$

- Viability kernel

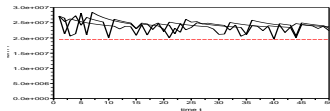
$$x \geq x_{pa} = 13775 \text{ tons}$$



Stock  $x(t)$



Catch  $c(t)$



Rent  $\Pi(t)$

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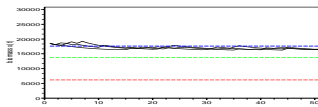
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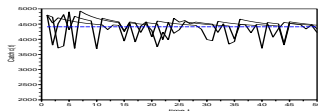
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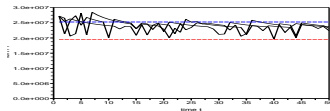
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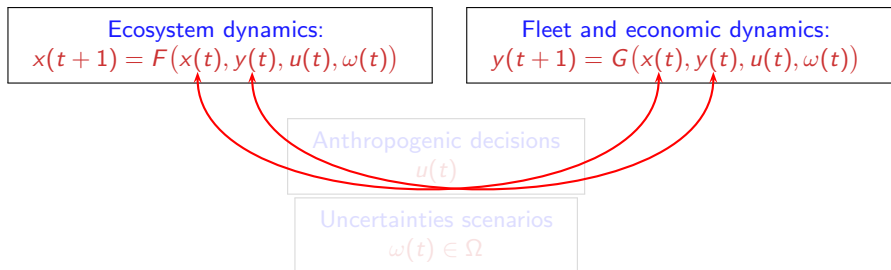
Catch  $c(t)$



Rent  $\Pi(t)$

# An abstract model

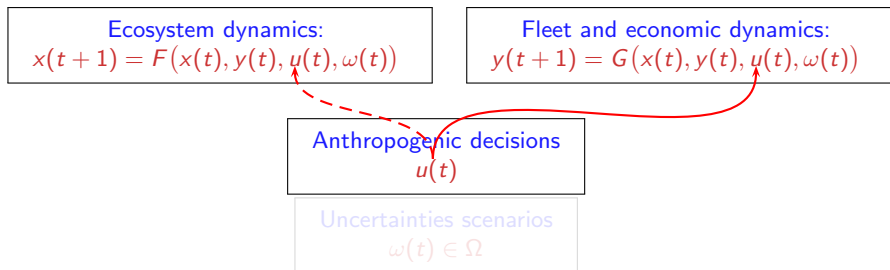
The framework: Control theory of noisy dynamic systems



- **Uncertainty**, stochasticity, scenarios, controversy

# An abstract model

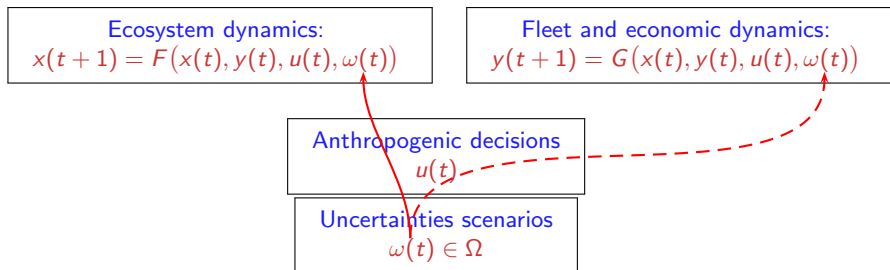
The framework: Control theory of noisy dynamic systems



- **Uncertainty**, stochasticity, scenarios, controversy

# An abstract model

The framework: Control theory of noisy dynamic systems



- **Uncertainty**, stochasticity, scenarios, controversy

# Co-viability constraints

Ecological Constraints:

Conservation

$$A(x(t)) \geq A_{\text{lim}}$$

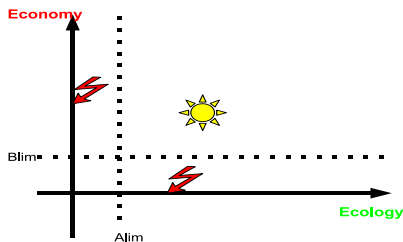
for  $t = t_0, \dots, T$

Economic constraints:

Utilities, rent, ..

$$B(x(t), y(t), u(t)) \geq B_{\text{lim}}$$

for  $t = t_0, \dots, T$

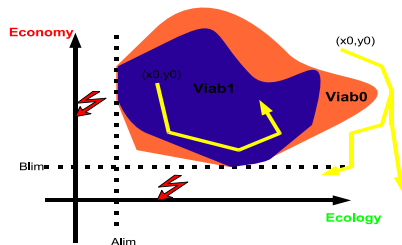


- **Multi-criteria:** Ecology, Economy
- **PVA:** a focus the ecological constraint
- **Intergenerational equity:** present & future

# Stochastic viability kernel

- Assume a probability  $\mathbb{P}$  i.i.d on  $\Omega$ .
- **Viability kernel** at confident rate  $\beta$ :

$$\text{Viab}_\beta(t_0) = \left\{ (x_0, y_0) \mid \exists u(\cdot) \text{ s.t. } \mathbb{P}_\omega \left( \text{constraints } t = t_0, \dots, T \right) \geq \beta \right\}.$$



- **Robust**: a particular case  $\beta = 1$ : no probability required
- **Irreversibility**: outside  $\text{Viab}_0(0)$

# Stochastic viability: a Dynamic programming approach

DeLara-Doyen, 2008

- Maximal viability probability before  $T$

$$V(t_0, x_0, y_0) = \max_{u(\cdot)} \mathbb{P} \left( (x(t), y(t), u(t)) \in \mathcal{K}, t = t_0, \dots, T \right)$$

- Value function by backward induction

$$\begin{cases} V(T, x, y) &= \mathbb{I}_{\mathcal{K}}(x, y) \\ V(t, x, y) &= \max_u \mathbb{E}_\omega \left[ \mathbb{I}_{\mathcal{K}}(x, y, u) * V(t+1, F(x, y, u), G(x, y, u)) \right] \end{cases}$$

where  $\mathbb{I}_{\mathcal{K}}(\cdot)$  characteristic function of  $\mathcal{K}$

# Viable decisions

- Feedback  $u(t, x, y)$ : adaptive
- Viable decisions:

$$u(t, x, y) \in \text{Arg max}_u \mathbb{E}_\omega \left[ \mathbf{I}_{K(t)}(x, y) * V(t + 1, F(x, y, u), G(x, y, u)) \right]$$

- No uniqueness: freedom
  - Conservative policy
  - Economics oriented policy
  - Trade-off

# Steady states, equilibrium and viability

- MSY
- **Equilibrium:** state  $(x^*, y^*)$  and decision  $u^*$  s.t.

$$x^* = F(x^*, y^*, u^*), \dots$$

- Equilibria are viable states

$$(x^*, y^*) \in \text{Viab}(t),$$

- Effort equilibria  $u^*$  are viable decisions

# Intergenerational equity, Maximin and viability

- Maximin  $\approx$  a particular ("extreme") case of viability

$$\max_{u(t)} \min_{t=0, \dots, T} B(x(t), y(t), u(t)) = \max_{(x_0, y_0) \in \text{Viab}(0)} B_{\text{lim}}$$

- Intergenerational equity of viability approach

## Other good news

- **Recovery, restoration**: Martinet-Thébaud-Doyen, 2007
- **Resilience**: Martin, 2006
- **Co-management, ITQ, coalition**: Eisenack et al., 2005;  
Doyen-Pereau, 2009
- **Precautionary approach & PVA**:  
Delara-Doyen-Rochet, 2007, Doyen-Pereau, 2008, Tichit et al., 2007
- **Sustainability, Maximin (intergenerational equity)**:  
Martinet-Doyen, 2007, Martinet, 2006, Baumgartner-Quaas, 2008
- **Ecosystem**  
Cury-Mullon-Garcia-Shanon, 2005; Doyen-DeLara-Pelletier-Ferraris,  
2007
- **Bounded rationality and "satisficing" H. Simon** Krawczyk, 2008

# Limits of the approach

- **Maths** are difficult especially in the stochastic case !!
- **Informatics**: Dynamic programming  
→ Curse of dimensionality !!!

# Another example

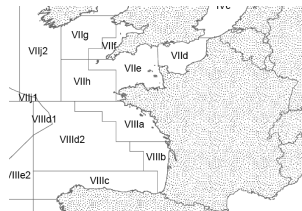
- Management of fisheries
- Stochastic case
- Simulation approach

# Bay of Biscay fisheries

Doyen-Thébaud-Béné-Bertignac-Fifas, Eco. Eco  
ANR biodiversity program



- **Multi-species:** nephrops, hake, ...
- **Multi-fleets** ICES Fishery Units :
- **By-catch** problem
- **Hake decline**
- **Calibration:** ICES 2006



# The exploited community dynamics: MSVPA

## Mortalities

$$N_{s,a}(t+1) = N_{s,a}(t) \exp \left( -M_{s,a} - \sum_{f=1}^K u_f(t) F_{s,a,f} \right),$$

where

- $s$  species,  $a$  age,  $f$  fleet
- $M_{s,a}$  natural mortality ;
- $F_{s,a,f}$  fishing mortality ;
- $u_f(t)$  fishing effort multipliers

## The catches

$$C_{s,a,f}(t) = N_{s,a}(t) u_f(t) F_{s,a,f} \times \frac{1 - \exp(-M_{s,a} - \sum_l u_l(t) F_{s,a,l})}{M_{s,a} + \sum_l u_l(t) F_{s,a,l}}$$

## Recruitment

$$N_{s,1}(t+1) = \varphi_s \left( SSB_s(N_s(t)), \omega(t) \right),$$

where

- $SSB_s$  spawning stock biomass of species  $s$

$$SSB_s(N_s) = \sum_{a=1}^A \gamma_{s,a} v_{s,a} N_{s,a},$$

- $\omega(t)$  the uncertainties  $\omega(t) \rightsquigarrow \mathcal{N}(\bar{B}_s, \sigma_s)$

## Income of fleet $f$

$$\text{Income}_f(t) = \sum_s p_{s,a} \sum_{a=1}^A w_{s,a} C_{s,a,f}(t) (1 - d_{s,a,f})$$

where

- $p_{s,a}$  the price of species
- $d_{s,a,f}$  discard

# Viability constraints and probabilities

Ecological Viability PVA

ICES precautionary approach

$$SSB_s(t) \geq B_s^{\text{lim}}$$

Economic Viability EVA

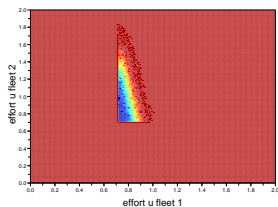
Sustainability of incomes

$$\text{Income}_f(t) \geq \text{Income}_f^{\text{lim}}$$

# Co-viability probability

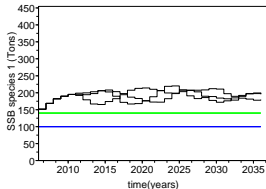
$\mathbb{P}(\text{constraints satisfied } t = 0, \dots, T)$

Co-viability probability

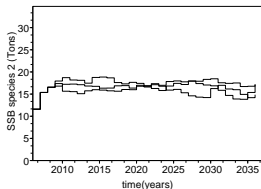


## Co-viability strategy $u_{CVA} = (0.72, 0.71)$

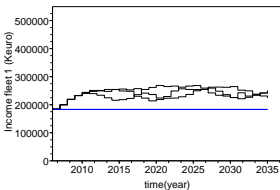
Viability with strategy  $u=(0.72; 0.71)$



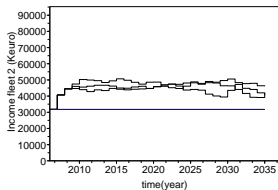
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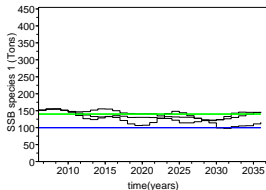
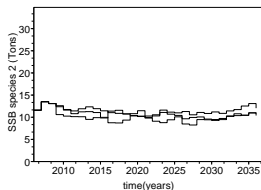
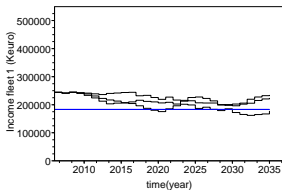
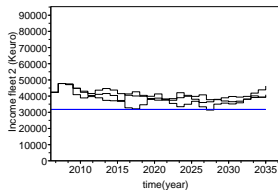
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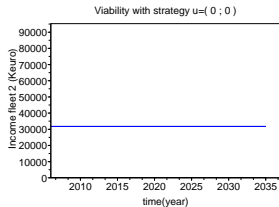
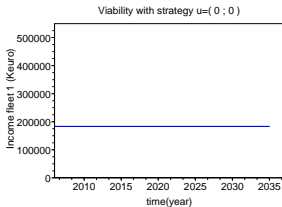
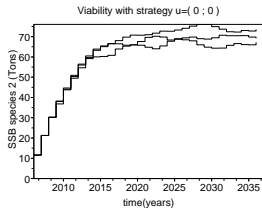
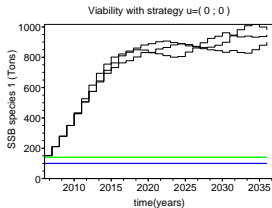
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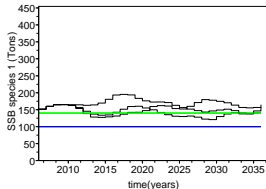
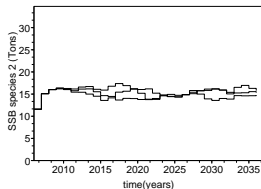
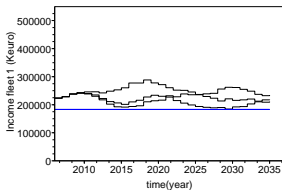
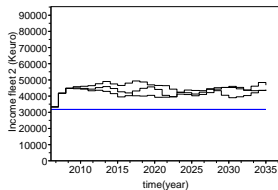
## 2006 baseline strategy $u_{BAU} = (1, 1)$

Viability with strategy  $u=(1; 1)$ Viability with strategy  $u=(1; 1)$ Viability with strategy  $u=(1; 1)$ Viability with strategy  $u=(1; 1)$ 

## Ecological strategy $u_{PVA} = (0, 0)$



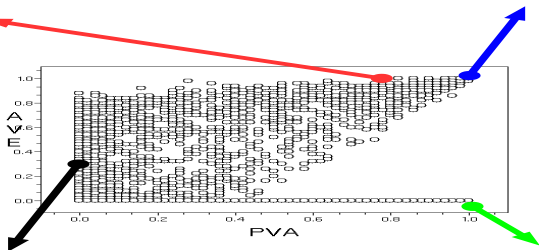
## Economic strategy $u_{EVA} = (0.85; 0.79)$

Viability with strategy  $u=(0.9 ; 0.75)$ Viability with strategy  $u=(0.9 ; 0.75)$ Viability with strategy  $u=(0.9 ; 0.75)$ Viability with strategy  $u=(0.9 ; 0.75)$ 

# Viable and non viable patterns

Economic strategy  $u_{EVA} = (0.85; 0.79)$

Co-viability strategy  $u_{CVA} = (0.72, 0.71)$   
 $\max_u \mathbb{P}(\text{Co-viability})$



2006 baseline strategy  $u_{BAU} = (1, 1)$

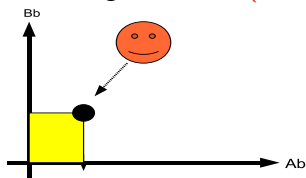
Ecological strategy  $u_{PVA} = (0, 0)$

# General conclusion and perspectives

- **Co-viability diagnostic:**
  - Interesting for sustainability and integrated diagnostic
- **Co-viability approach:**
  - between conservation biology and bio-economics
- **Co-viability mathematics:**
  - between equilibrium and optimality under constraints
  - risk management

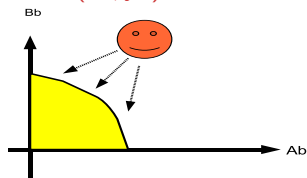
# An inverse problem

Given initial bio-economic state  $(x_0, y_0)$ ;  
Which guaranteed  $(A_{lim}, B_{lim})$  such that  $(x_0, y_0) \in Viab??$



Win-win

or



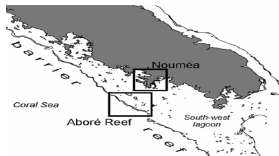
or

Trade-off ??

# Management through protected areas

Doyen-DeLara-Pelletier-Ferraris, 2007

- A trophic web:  
**Piscivors, Carnivors,  
Herbivors, ...**
- Habitat dynamics: Coral
- Uncertain cyclonic event
- Strong catch pressure
- Some data in  
New-Caledonian



# Dynamic model

## Trophodynamics

$$x_i(t+1) = x_i(t) \left( R + \exp(y_1^\# - y_1(t)) Sx(t) \right)_i - c_i(t)$$

- trophic web matrix  $S$

$$S = \begin{pmatrix} -0.093 & 0.013 & 0.013 & 0.013 \\ -0.106 & -0.012 & 0.002 & 0.002 \\ -0.076 & -0.01 & 0. & 0. \\ -0.53 & -0.069 & 0. & 0. \end{pmatrix}$$

- growth  $R = (0.975, 1.007, 1.008, 1.054)'$
- A refuge effect through coral  $y_1(t)$  covering:

$$\exp(y_1^\# - y_1(t))$$

- Gordon Schaefer production

$$c_i(t) = q_i(1 - \text{MPA})e(t)x_i(t)$$

## Coral dynamics

$$y_1(t+1) = y_1(t) \cdot \begin{cases} R_{\text{COR}} \left( 1 - \frac{y_1(t)}{K_{\text{COR}}} \right) & \text{with proba}(1 - p) \\ 0.3 & \text{with proba } p \end{cases}$$

where

- $p = 1/5 * 365$  probability of a cyclonic event.
- $R_{\text{COR}} = 1.002$  intrinsic productivity at low cover levels.
- $K_{\text{COR}}$  carrying capacity

$$1 = R_{\text{COR}} \left( 1 - \frac{y_1^*}{K_{\text{COR}}} \right), \quad y_1^* = \frac{R_{\text{COR}} - 1}{R_{\text{COR}}} K_{\text{COR}} = 0.8.$$

# Co-viability

## Direct use

$$U(c_1(t), c_2(t), c_3(t)) \geq U_{\text{lim}}$$

where

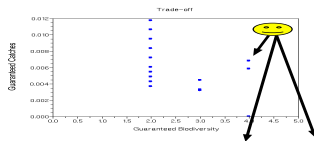
- $U_{\text{lim}} > 0$  guaranteed utility
- $U(c_1, c_2, c_3) = w_1 c_1 + w_2 c_2 + w_3 c_3$   
where  $w_i$  mean weight of group  $i$ .

## Conservation

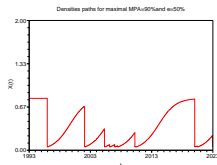
A trophic richness is guaranteed

$$I(x(t)) = \sum_i \mathbf{1}_{\mathbb{R}^*_+}(x_i(t)) \geq I_{\text{lim}}.$$

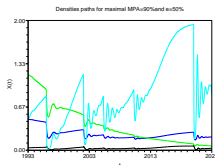
# Trade-off



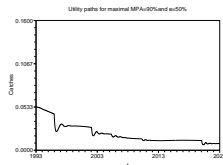
MPA = 90%



Coral  $x_5(t)$



Fish  $x(t)$



Catch weight  $c(t)$