Design of Sustainable Quotas for an Hake–Anchovy Peruvian Ecosystem Model by Viability Methods

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September 8, 2009

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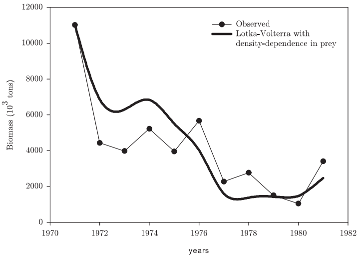
Anchoveta y Merluza

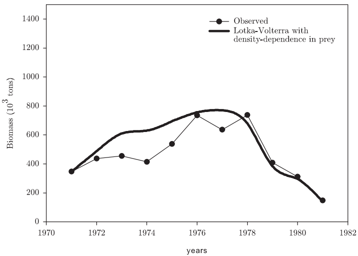
11 years of data from 1971 to 1981

In thousands of tonnes (10^3 tons)

- anchoveta_stocks= [4058 3116 3461 2649 4517 1232 3727 1812 1826 8793 3418]
- merluza_stocks= [347 437 455 414 538 735 636 738 408 312 148]
- anchoveta_captures= [5797 1600 2540 3191 2299 1323 353 1154 177 202 1209]
- merluza_captures= [27 13 133 109 85 93 107 303 93 159 69]



(c) Anchovy



(d) Hake

Figure: Comparison of observed and simulated biomasses of anchovy and hake using a Lotka–Volterra model with density-dependence in the prey. Model parameters are $R = 2.24$, $L = 0.98$, $\kappa = 64\,672 \times 10^3$ t ($K = 35\,800 \times 10^3$ t), $\alpha = 1.230 \times 10^{-6}\ t^{-1}$, $\beta = 4.326 \times 10^{-8}\ t^{-1}$.

Conservation and catch thresholds

	Anchovy (prey, y)	Hake (predator, z)
minimal biomass	7 000 kt	200 kt
minimal catch	2 000 kt	5 kt

These annual objectives were theoretically jointly achievable but. . .

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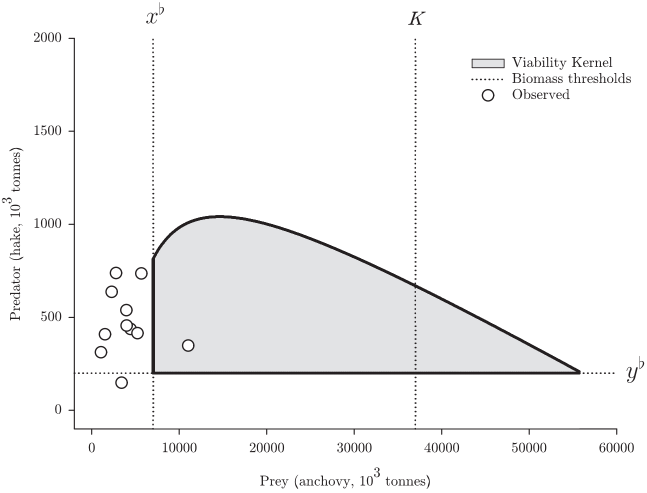

$x^\flat$
K
Viability Kernel
Biomass thresholds
Observed
2000
1500
1000
500
0
Predator (hake, 10^3 tonnes)
0
10000
20000
30000
40000
50000
60000
$y^\flat$
Prey (anchovy, 10^3 tonnes)

Lotka–Volterra Model With Density–Dependence

$$\begin{cases} y(t+1) = y(t)\underbrace{\left(R - \frac{R}{\kappa}y(t) - \alpha z(t) - v(t)\right)}_{R_y}, \\ z(t+1) = z(t)\underbrace{\left(L + \beta y(t) - w(t)\right)}_{R_z}, \end{cases}$$

- state vector (y, z) represents biomasses,
  - y prey biomass: anchovy
  - z predator biomass: hake
- control vector (v, w) is fishing effort of each species,
- catches are vy and wz (measured in biomass),
- R_y and R_z are annual growth factors.

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Viability kernel

C. Béné, L. Doyen, and D. Gabay. *A viability analysis for a bio-economic model.* Ecological Economics, 36:385–396, 2001.

The viability kernel is the set of initial states $(y(t_0), z(t_0))$ from which can emerge a trajectory $(y(t), z(t))$, $t = t_0, t_0 + 1, \ldots$ driven by appropriate controls $(v(t), w(t))$, $t = t_0, t_0 + 1, \ldots$ such that the following goals are satisfied

- preservation (minimal biomass thresholds)

$$\text{stocks:} \qquad y(t) \geq y^\flat \;, \quad z(t) \geq z^\flat$$

- and economic/social requirements (minimal catch thresholds)

$$\text{catches:} \qquad v(t)y(t) \geq Y^\flat \;, \quad w(t)z(t) \geq Z^\flat \;.$$

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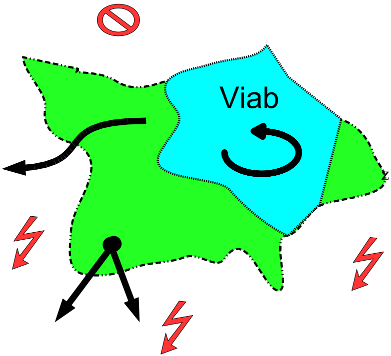

Figure: The state constraint set is the large set. It includes the smaller viability kernel.

Explicit expression for the viability kernel

Proposition

- *If the growth factors are decreasing in the fishing effort*
- *and if the thresholds are such that the following growth factors are greater than one*

$$R_y(y^\flat, z^\flat, \frac{Y^\flat}{y^\flat}) \geq 1 \ \text{and} \ R_z(y^\flat, z^\flat, \frac{Z^\flat}{z^\flat}) \geq 1 \ ,$$

the viability kernel is given by

$$\left\{ (y,z) \mid y \geq y^\flat, \ z \geq z^\flat, \ yR_y(y,z,\frac{Y^\flat}{y}) \geq y^\flat, \ zR_z(y,z,\frac{Z^\flat}{z}) \geq z^\flat \right\}.$$

Hence, for given thresholds $y^\flat, z^\flat, Y^\flat, Z^\flat$, we can tell whether or not they can be indefinitely maintained starting from (y,z).

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Adjusting Catches to Prominent Biomass Conservation Thresholds

1. Considering that first are given minimal biomass conservation thresholds

$$y^\flat \geq 0 \;, \quad z^\flat \geq 0$$

2. and defining

$$\begin{cases} Y^{\flat,\star} & := & y^\flat \max\{v \geq 0 \mid R_y(y^\flat, z^\flat, v) \geq 1\} \\ Z^{\flat,\star} & := & z^\flat \max\{w \geq 0 \mid R_z(y^\flat, z^\flat, w) \geq 1\} \end{cases}$$

3. the following catches levels $Y^\flat$ and $Z^\flat$ are susceptible to be sustainably maintained starting from $y \geq y^\flat$ and $z \geq z^\flat$:

$$\begin{cases} Z^\flat & \leq & Z^{\flat,\star} \\ Y^\flat & \leq & \min\left\{Y^{\flat,\star}, y(R - Ry/\kappa - \alpha z) - y^\flat\right\} \;. \end{cases}$$

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Hake–anchovy Peruvian fishery: Peru official quotas and sustainable quotas given by the viability approach

	Sustainable quotas (kt)		Peru official quotas (kt)	
	Model 1	Model 2	2006	2007
Anchovy	5 152	5 399	4 250	5 300
Hake	49	$56,8$	55	35

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- Managing ecological and economic conflicting objectives
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