Sustainable coalitions in the commons

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Motivations

Which complex balance between
resource dynamics and strategic interactions
to avoid bio-economic collapses ?

- How to combine game theory and viability approach ?
- Shape of viable coalition formed by heterogeneous users ?
- Marginal contribution of users to the sustainability ?

Related literature :

- Hardin : "tragedy of commons"
- Non cooperative game theory
  - Mesterton-Gibbons (NRM, 1993)
  - Sandal and Steinshamn (JEDC, 2004)
- Coalition game theory
  - Cooperative game : Lindroos (2004), Konbak-Lindroos (2007)
  - Endogenous coalition formation : Pintassilgo (NRM, 2003)

The exploited renewable resource

- Dynamics of the exploited resource $x(t)$

$$x(t+1) = f\Big(x(t) - h(t)\Big)$$

- Catches by n agents

$$h(t) = \sum_{i=1}^{n} q_i e_i(t) x(t)$$

with e_i effort of agent i

The agents exploiting the resource

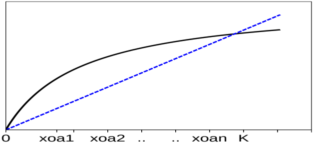
- n heterogeneous agents : c_i cost of effort and q_i catchability
- Rent of agent i with p price

$$\Pi_i(x(t), e_i(t)) = pq_i e_i(t) x(t) - c_i e_i(t)$$

- Open-access levels $x_{\text{OA}_i} = \frac{c_i}{pq_i}$

We assume

$$x_{\text{OA}_1} < x_{\text{OA}_2} < .... < x_{\text{OA}_n} < K$$



Sustainability issue

- Bio-economic goals : find
  - coalitions of agents : $S \subset \{1, \dots, n\}$
  - harvesting strategies among the coalition $e_i(t),\ i \in S$
  - a stock path $x(t)$

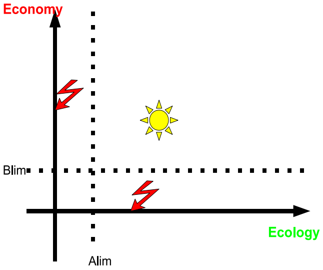
  which ensure a positive aggregated rent for the coalition

$$\sum_{i \in S} \Pi_i(x(t), e_i(t)) > 0, \qquad t = 0, 1, ..., T$$

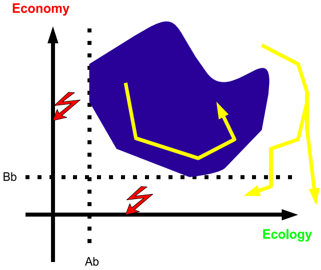
- A necessary ecological viability condition

$$x(t) > x^S = \min_{i \in S} x_{\mathrm{OA}_i}$$

Viability approach



Constraints



Viability kernel

Viability approach

- Viability kernels $\mathrm{Viab}_S(t)$ for a coalition S ? ? ?
- Dynamic programming :
  - At the terminal date T

$$\mathrm{Viab}_S(T) = \{x \mid x > x^S\}$$

  - For any time $t = 0, 1, ..., T-1$

$$\mathrm{Viab}_S(t) = \left\{ x > x^S \;\middle|\; f\left(x(1 - \sum_{i \in S} q_i e_i - \sum_{j \notin S} q_j e_j)\right) \in \mathrm{Viab}_S(t+1) \right\}$$

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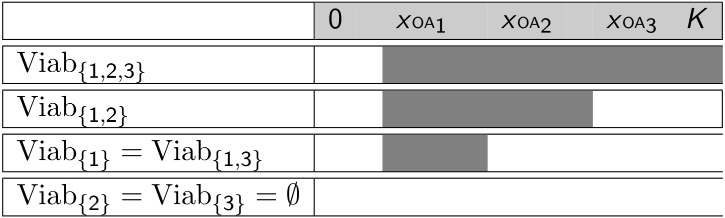
Example : $n = 3$: The tragedy of open-access revisited



Coalition singletons :
the smallest viability ! ! !

$$\mathrm{Viab}_{\{2\}}(t) = \mathrm{Viab}_{\{3\}}(t) = \emptyset$$

$$\mathrm{Viab}_{\{1\}}(t) =]x_{OA1}, x_{OA2}[$$

Grand Coalition :
the largest viability ! ! !

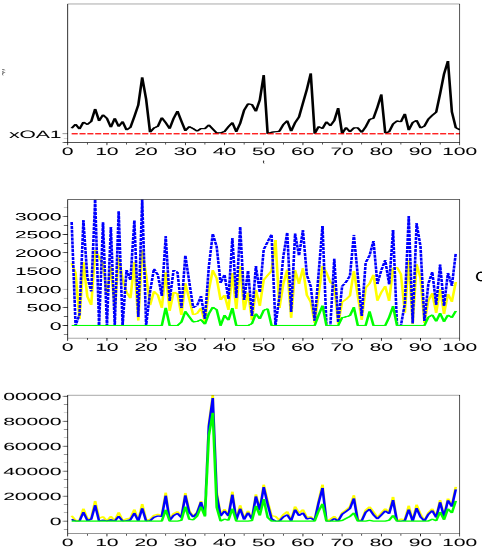
$$\mathrm{Viab}_{\{1,2,3\}}(t) =]x_{OA1}, +\infty[$$

- Largest viability

  $\mathrm{Viab}_{\{1,2,3\}} =]x_{\mathrm{OA1}}, +\infty[$

- MEY viable

  $x_{\mathrm{MEY}_i} > x_{\mathrm{OA1}} \in \mathrm{Viab}_S$



Stock $x(t)$

Effort of agents $e_i(t)$

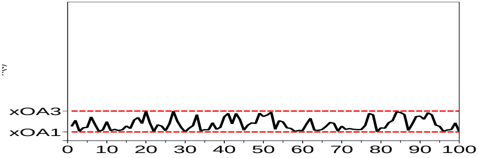
Rent of agents $\Pi_i(t)$

A focus on the partial coalition $S = \{1, 2\}$

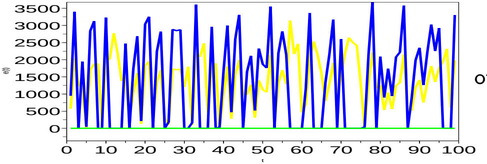
- Reduced viability

$$\mathrm{Viab}_{\{1,2\}} =]x_{OA1}, x_{OA3}[$$

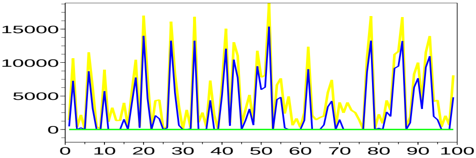
- Agent 3 is neutralized by 1 and 2



Stock
$x(t)$



Effort
of agents
$e_i(t)$



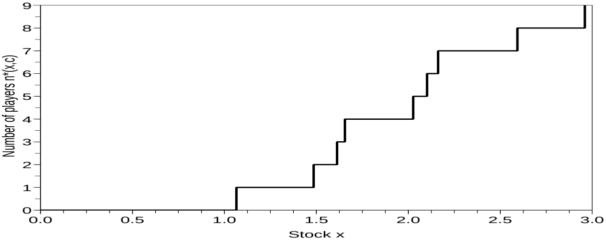
Rent
of agents
$\Pi_i(t)$

Minimum number of agents in a viable coalition

- Extension of `Sandal-Steinshamn, 2004` (equilibrium) :

$$n^*(x) = \min\left(|S| \mid x \in \mathrm{Viab}_S(0)\right)$$
$$= \min\left(j \mid x_{\mathrm{OA}_1} < x < x_{\mathrm{OA}_{j+1}}\right\}$$

where $|S|$ the cardinal of coalition S



- The number of viable agents increases with the stock ! ! !

Marginal contribution to viability

- Marginal contribution of agents $i \in S$ to the viability kernel : Shapley value

$$Sh_i(x) = \sum_{i \in S \subseteq N} \frac{(|S|-1)!(n-|S|)!}{n!} \left(\mathbf{1}_{\mathrm{Viab}_S}(x) - \mathbf{1}_{\mathrm{Viab}_{S\setminus\{i\}}}(x) \right)$$

- Application $n = 3$

Agents $i \backslash$ Stock x	0	x_{OA1}	x_{OA2}	x_{OA3}
agent 1	0	100%	50%	33%
agent 2	0	0	50%	33%
agent 3	0	0	0	33%

- Agent 1 always veto agent
- Agent 1 dictator if resource low
- Equity between active agents

Viability kernels in the general case

Theorem

The viability kernels at time t are

- $\mathrm{Viab}_S(t) = \emptyset$ *if* $1 \notin S$
- $\mathrm{Viab}_{\{1,\ldots,n\}}(t) =]x_{OA1}, +\infty[$
- $\mathrm{Viab}_{\{1,\ldots,j\}}(t) = \left] x_1^{\infty}, x_{j+1}^{\infty} \right[$ *for* $j < n$
- $\mathrm{Viab}_S(t) = \mathrm{Viab}_{\widetilde{S}}(t)$ *for* $\widetilde{S} = \cup_i \left(\{1 \ldots i\} \subset S \right)$

Viable efforts for the viable coalitions

- The viable feedbacks $e^*_S(t,x) = (e^*_i(t,x))_{i\in S}$ for a coalition S and any stock x in $\mathrm{Viab}_{\{1\ldots j\}}(t) = \left]x_1^\infty, x_{j+1}^\infty\right[$ are solutions of the linear constraints
  - $\sum_{i\in S} e^*_i(px - c_i) > 0$
  - $\sum_{i\in S} e^*_i(t,x) =$
    $\alpha\left(1 - \frac{f^{-1}(x_1^\infty)}{x}\right) + (1-\alpha)\max\left(0, 1 - \frac{f^{-1}(x_{j+1}^\infty)}{x}\right)$
    with $0 < \alpha < 1$.
- Flexible decisions :
  - Ecological and economic performances
  - Passive outsiders

A "simple" game formulation

- Maxmin formulation

$$\begin{cases} V(T,x) & = & \mathbf{1}_{\{x > x^S\}}(x) \\ V(t,x) & = & \displaystyle\sup_{\begin{cases} e_i, \ i \in S \\ e_i \geq 0, \\ \sum_{i \in S} \Pi_i(x,e_i) > 0 \end{cases}} \ \inf_{\begin{cases} e_j, \ j \notin S, \\ e_j \geq 0 \\ \Pi_j(x,e_j) \geq 0 \end{cases}} V\left(t+1, f\left(x-h\right)\right) \end{cases}$$

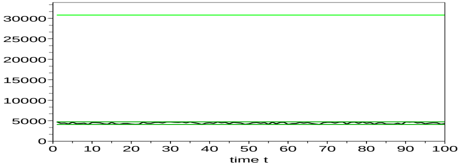
- Result : V Indicator function of the viability kernel Viab_S

$$V(t,x) = \mathbf{1}_{\mathrm{Viab}_S(t)}(x) = \begin{cases} 1 & \text{if} & x \in \mathrm{Viab}_S(t) \\ 0 & \text{if} & x \notin \mathrm{Viab}_S(t). \end{cases}$$

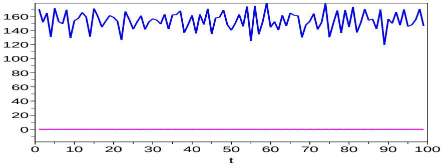
A focus on the singleton $S = \{1\}$

- Smallest viability

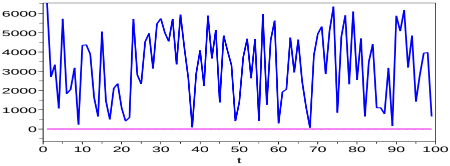
  $$\mathrm{Viab}_{\{1\}} =]x_{OA1}, x_{OA2}[$$



Stock
$x(t)$



Effort
of agents
$e_i(t)$



Rent
of agents
$\Pi_i(t)$

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$$\mathrm{Viab}_{\{1\}} =]x_{\mathrm{OA}1}, x_{\mathrm{OA}2}[$$

More generally

Result : Equal shares for active agents :

For $x \in \left] x_j^\infty, x_{j+1}^\infty \right[$:

$$Sh_i(x) = \begin{cases} \frac{1}{n^*(x)} & \text{for} \quad i \leq j \\ 0 & \text{for} \quad i > j \end{cases}$$

Related literature

- Mesterton-Gibbons

$$max_{e_i}\Pi_i = (pq_i xss(\sum_{i=1}^{n} e_i) - c_i)e_i$$

$$n^* = max(i \mid ib_i < 1 + \sum_{j=1}^{i-1} b_j)$$

with $b_i = c_i / Kpq_i$

- Sandal and Steinshamn
  - Cournot competition

$$max_{h_i}\Pi_i = (p(\sum_{i=1}^{n} h_i) - c_i(x))h_i$$

  - solution $h_i^* = h_i(x)$
  - then substitute $x^{ss} = g(\sum_{i=1}^{n} h_i)$